Pedagogy lecture

Performance of wireless Communication Systems with imperfect CSI

Yogesh Trivedi
Associate Prof.

Department of Electronics and Communication Engineering
Institute of Technology
Nirma University

July 1, 2011
Outline

- Performance of wireless system with perfect CSI in SISO systems
- Enhancing performance using MIMO systems with perfect CSI
- Causes of Imperfect CSI.
- Performance of MIMO systems with imperfect CSI.
- Future scope
Performance Analysis of SISO systems with perfect CSI

- Here channel is modelled by Rayleigh distribution.

- Received Symbol $y = hx + n$, where $x$ is a transmitted symbol of average power $E_s$, whereas $n \sim CN(0, N_0)$ and $h \sim CN(0, 1)$.

- Coherent detection at the receiver: The sufficient statistics or decision variable $z$ for $x$ is

$$z = \frac{h^*}{|h|} y = |h|x + n'$$

- pdf of SNR $\gamma$ is exponential

$$p(\gamma) = \frac{1}{\gamma_c} e^{-\frac{\gamma}{\gamma_c}}, \quad \text{where } \gamma_c = \frac{E_s}{N_0} \quad (1)$$

- BER can be derived as

$$P_e = \int_0^\infty Q(\sqrt{2\gamma})p(\gamma)d\gamma = \frac{1}{2} \left(1 - \sqrt{\frac{\gamma_c}{\gamma_c + 1}}\right)$$
Performance of wired and wireless systems
Use of Diversity in wireless systems

Different types of diversity

- **Time diversity**: Symbol is transmitted in multiple time-slots, which reduces data rate. e.g. Coding and interleaving

- **Frequency diversity**: Symbol is transmitted in multiple frequencies, which increases bandwidth.

- **Space diversity**: Symbol is transmitted through multiple antennas.
  - **Receive diversity**: SIMO systems
  - **Transmit diversity**: MISO systems
  - **Transmit-Receive diversity**: MIMO systems
Receive Diversity

Different types of Combining

- **Maximum Ratio Combining (MRC):** Perfect CSI is required.
- **Equal Gain Combining (EGC):** Only the phase of the CSI is required.

Performance of MRC with $N$ receive antennas

$$P_e = \left(\frac{1 - \mu}{2}\right)^N \sum_{p=0}^{N-1} \binom{N-1+p}{p} \left(\frac{1 + \mu}{2}\right)^p$$

where $\mu = \sqrt{\frac{E_s}{E_s+N_0}}$
MRC with multiple receive antennas (N)
Transmit Diversity (Space Time Code)

- Space Time Block Codes: In which $2 \times 1$ Alamouti code is very popular.

$$
\begin{bmatrix}
  r_1 \\
  r_2^*
\end{bmatrix}
= 
\begin{bmatrix}
  h_1 & h_2 \\
  h_2^* & -h_1^*
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2^*
\end{bmatrix} + 
\begin{bmatrix}
  n_1 \\
  n_2^*
\end{bmatrix}
$$

- Decision variable for data-symbol $x_1$ is $Re\{z_1\}$, where

$$
z_1 = [h_1^* \ h_2]
\begin{bmatrix}
  r_1 \\
  r_2^*
\end{bmatrix}
$$

- Diversity gain of this $2 \times 1$ system is same as $1 \times 2$ MRC system.
Comparision between MRC and Alamouti systems

![Graph comparing BER vs Avg. SNR dB for Alamouti System (2x1) and MRC system (1x2)]
Transmit Diversity (Transmit Beamforming) for $N \times 1$

- The received signal is $r = hwx + n$.
  - $w$ is the beamforming vector of $N \times 1$, which is $h^*/\|h\|$.
  - $x$ is the data symbol with average power $E_s$.
  - All the coefficients of $h$ are i.i.d. as complex normal with mean zero and variance one.

- At the receiver, the decision of the transmitted symbol is given by the sign of the decision variable

$$z = \text{Re}\{r\}$$
Transmit Beamforming-MIMO systems

- Transmitter and receiver have $N_t$ and $N_r$ antennas respectively.

- Received symbol $r = Hw x + n$, where $H$ is $N_r \times N_t$ channel matrix, $r$ and $n$ are received symbols and AWGN respectively and each is of $N_r \times 1$, $w$ is a unit beamforming vector of order $N_t \times 1$.

- To maximize the received SNR $w$ should be the eigenvector corresponding to the maximum eigenvalue of $H^*H$.

- At the receiver, the decision of the transmitted symbol is given by the sign of the decision variable

$$z = \text{Re}\{(Hw)^*r\}$$

- It provide diversity gain of order $N_t \times N_r$. 

Drawbacks of MIMO systems

The performance of a MIMO system enhances with more number of transmit antennas. However there are some drawbacks in its realization.

- The cost of implementing multiple RF circuits.
- It is difficult to install multiple antennas for a mobile device due to smaller size, insufficient spacing between adjacent antennas and increased price.
- For Space time coded systems, the complexity of decoder will increase with more antennas.
- Power spreading between multiple antennas results in loss of SNR.
Antenna selection

An effective technique to reduce the cost and the complexity of MIMO systems is Antenna Selection.

- With AS, a reduced number of RF chains are used still providing full diversity benefits.
- In case of optimum AS, the complexity of algorithms and the number of feedback bits are more.
- Therefore, sub-optimum AS schemes are of interest.
Selection Combining

In SIMO systems, two out of $N$ antennas are selected and then combined by MRC $(1; N, 2)$. 
Channel estimation at the receiver

Central to exploit antenna diversity is availability of perfect CSI.

- Channel is estimated by sending high energy Pilot or training symbols.
  - increases overheads
  - reduces data rate and spectral efficiency.
  - for estimation of stationary channels (for example by Wiener filter), statistical information is also to be estimated.
  - For estimation of non-stationary channels (for example by Kalman filter), complex signal processing is required.
- Time-varying nature of wireless channel makes the scenario even worse.
- Non-ideal (noisy and delayed) feedback link

- All these factors lead towards erroneous or imperfect CSI at the receiver. Therefore, performance analysis of wireless systems with imperfect CSI is of interest.
Performance of SISO systems with imperfect CSI at the receiver

Let imperfect CSI at the receiver be \( \hat{h} \) and the correlation between \( h \) and \( \hat{h} \) is \( \rho \) i.e. \( E[h^*\hat{h}] = \rho \), where \( 0 \leq \rho \leq 1 \). Then, using first order Gauss-Markov model

\[
h = \rho \hat{h} + \sqrt{1 - \rho^2} w, \tag{1}
\]

where \( w \sim CN(0, 1) \) and it is independent of \( \hat{h} \). Furthermore, the correlation coefficient \( \rho = 0 \) represents no CSIR, whereas \( \rho = 1 \) represents perfect CSIR.

Now, using this \( \hat{h} \) at the receiver, the decision variable \( d \) is

\[
d = Re \left\{ \hat{h}^* y \right\}
\]

and

- if \( d \geq 0 \), the detected symbol is 1 otherwise detected symbol is 0.

\[
P_e = \frac{1}{2} \left( 1 - \rho \sqrt{\frac{E_s}{E_s + N_0}} \right)
\]
Performance of SISO systems with imperfect CSI at the receiver

![Graph showing the performance of SISO systems with imperfect CSI at the receiver. The graph plots BER against average SNR (E_s/N_0) in dB. The y-axis represents BER ranging from 10^-4 to 10^0, and the x-axis represents average SNR ranging from 5 to 25 dB. The graph includes lines for analytical results, simulation results, and perfect CSI, with different correlation coefficients (\(\rho\)) indicated: \(\rho=0.995\), \(\rho=0.999\), and \(\rho=0.99\).]
Performance of TB-MISO systems with imperfect CSI at the transmitter

Let imperfect CSI at the transmitter be \( \hat{h} \) and the correlation between \( h \) and \( \hat{h} \) is \( \rho \) i.e. \( E[h^*\hat{h}] = \rho \), where \( 0 \leq \rho \leq 1 \). Then, using Gauss-Markov model, we can represent \( h \) as

\[
 h = \rho \hat{h} + \sqrt{1 - \rho^2} v,
\]

(3)

where \( v \) is independent of \( \hat{h} \).

For this system, BER can be expressed as

\[
P_e = (1 - \rho^2) \left[ \frac{1}{2} (1 - \mu) \right] + \rho^2 \left[ \frac{1}{4} (2 - 3\mu + \mu^3) \right],
\]

where

\[
\mu = \sqrt{\frac{E_s}{E_s + N_0}}
\]

(4)
Performance of TB-MISO systems with imperfect CSI at the transmitter

Figure 1: BER Vs Avg. SNR for BPSK
Performance of TB-MISO systems with antenna selection using imperfect CSI at the transmitter in $(3, 2; 1)$ system

"Performance analysis of TB MISO systems with antenna selection using delayed CSI at the transmitter" by Y N Trivedi and A K Chaturvedi, IET Communications, April 2011.
Performance of Space Time Coded-MISO systems with antenna selection using imperfect CSI at the transmitter in $(3, 2; 1)$ system

![Graph showing BER vs Avg. SNR dB for different correlation coefficients $\rho$. The graph includes curves for $\rho = 0.01$, $\rho = 0.8$, and $\rho = 0.97$, with simulations marked by asterisks.](image-url)
Future scope

- We have considered Gaussian channels, however more general like Rician or Nakagami channels can be considered.

- We have also considered imperfection in CSI as Gaussian only which may be in other forms also, for example quantized CSI.

- We have assumed spatially uncorrelated channels, however in practice it may be correlated also.

- The considered system models can also be analyzed by taking OFDM.